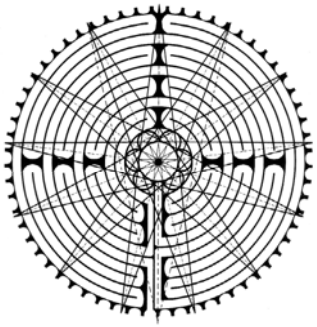
Division of the circle into thirteen: A dialogue

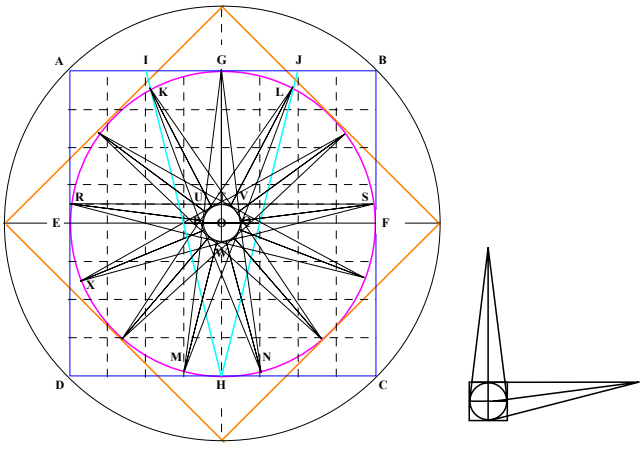
Lee Dickey and Vanessa Compton



The 13 Pointed Star

Critchlow's diagram of the 13 pointed star, in Lauren Artress' book *Walking a Sacred Path* (1995), which he claimed underlay the rosette pattern in the centre of the Chartres labyrinth, provoked our search for the geometrical process behind it.

Vanessa's first attempt



Construction of the thirteen point star using the acute and right angle triangles

Inscribe a circle at centre O to fill completely a square ABCD. Lay in central vertical and horizontal axes of the square, EF and GH, to intersect at O.

Construct an inverted acute angle triangle IHJ from the centre point H of the bottom line to the two quarter marks I and J on the top line. The points K and L where the edges of the triangle intersect the circle's circumference mark the vertices of the two arms of the star on either side of the central vertical axis GH.

Construct a circle one eighth the diameter of the original circle, with the same centre O.

Lines drawn from point G to line DC, tangent to the small circle at P and Q, will intersect the large circle's circumference at the points M and N, the intersections of the one eighth vertical lines to the left and right of the centre vertical axis GH. MGN is the first arm of the star, with centre line OG.

A horizontal line RS drawn tangent to the top of the small circle is one sixteenth of the large circle's diameter in distance from the centre O, and intersects GH at point T. Construct right angle triangles GTU and GTV where U is the intersection of GM and RS, V is the intersection of GN and RS.

GH is perpendicular to RS, giving right angles GTR and GTS. Similarly, GN intersects SO at right angle GQS and NQS. This relationship between the edges of the star's arms and their centre lines radiant from O exists between every third arm. Line ON gives a right angle SWN at W tangent to the small circle. SW can be extended to intersect the large circle at X, creating another arm of the star with centre line OS and marking the point of another arm with centre line XO. This process may be continued until all thirteen of the star's arms are generated.

Lee Dickey: This has an accumulated error of 4.96°

Dr. Lee Dickey's Constructions of the 13-gon

Email Tuesday Nov 11.97
I have prepared four nice pictures for you, based on four constructions that I will list at the end of this message. There is a "base figure" common to all three, the circle inscribed in a square. I have adopted a North, East, South, and West labels for the basic four directions of the circle.

I am trying to create a language for documenting Cabri steps. I want a person (like you, for instance) to be able to read this and turn it into a Cabri construction or even a straightedge and compass construction. My goal is to have it

(1) crystal clear
(2) compact

In my stuff that follows, I am trying out my current version of this language. I would be interested in your reaction. Please don't puke.

Back to the 13-gon.
I have done three angles, alpha, beta, and gamma, each better than the one before by a factor of 50 to 100. All three use a common "Base Figure". The Base figure has the circle and the square around it. I think that alpha+ appears somewhere in your work, or maybe it is similar to something I thought I saw there. The beta is my first stab at getting 13*alpha within tenth of a degree of 360. It is off by about 1/40 of a degree.
The Gamma is an improvement. It is accurate to about 1/2000 of a degree, but it has the disadvantage of being more cumbersome to do.

Lee
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The Base figure

Let O, E be any points.
Z = circle (O, E)
L1 = line (O,E)
{E,W} = meet (L1, Z)
L2 = perp (L1, O).
L3 = perp (L2, N)
L4 = perp (L1, E)
L5 = perp (L2, S)
L6 = perp (L1, W).
A = meet (L3, L6)
B = meet (L3, L4)
C = meet (L4, L5)
D = meet (L5, L6)
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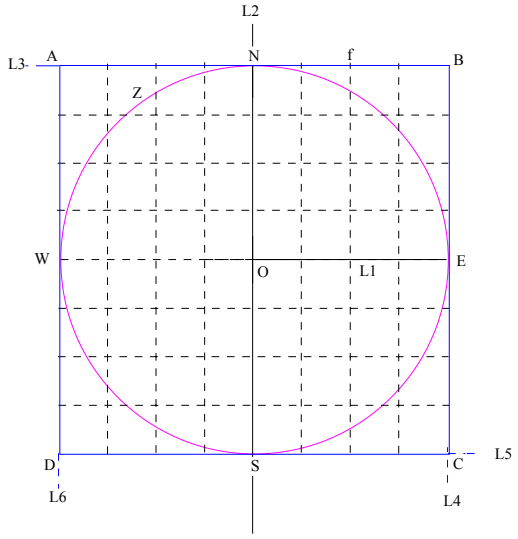


The Base Figure

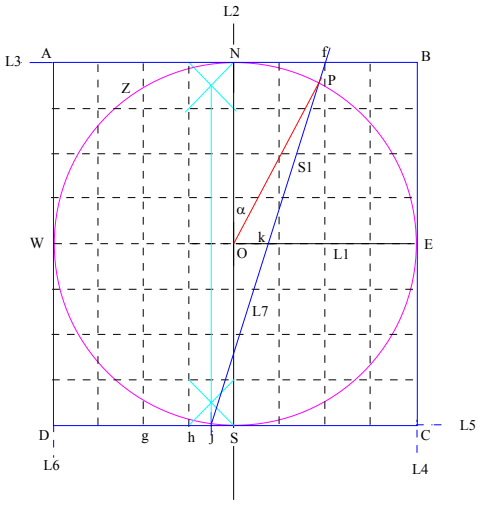
Construction for alpha

Use the "Base Figure",

f = mid (N, B)
g = mid (D, S)
h = mid (g, S)
j = mid (h, S)
L7 = line (f, j)
k = meet (L1, L7)
S1 = segment (k, f)
P = meet (S1, Z)
alpha = < (N, O, P)

(k is used to define a segment that meets Z in only one point)

alpha = 27.663 504 190 001 236 °
13 * alpha = 359.625 554 470 016 064°
Rel. Error = 0.104 012 647 217 8 %
-+-+-+-+-+-+-+-+-+-+-+-+-+



The Base Figure and alpha

Construction for beta

Use the Base Figure

f = mid (N, B)

g = mid (O, S)
h = mid (g, S).
j = midpt (g, h).
S1 = segment (j, f)
Q = meet (S1, Z)
beta = < (N, O, Q)

beta = 27.694 119 871 484 227°
13 * beta = 360.023 558 329 294 945°
Rel. Error = 0.006 543 980 359 7 %
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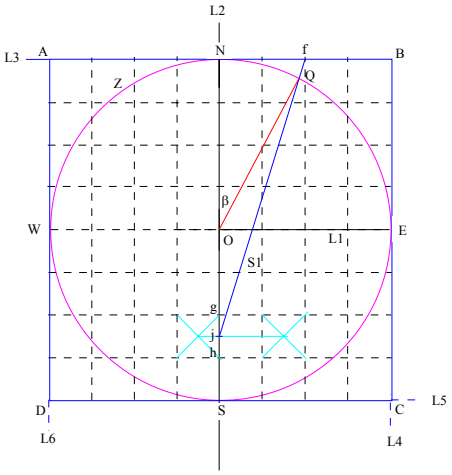

The Base Figure and beta

Construction for gamma

Starting with the Base Figure:

f = mid (N, B)
g = mid (D, S)
h = mid (g, S)
j = mid (h, S)

L7 = bisect (< (E, O,C))

k = meet (L4, L7)
R1 = ray (0, k)
m = meet (R1, Z)

S2 = segment (j, m)
n = meet (S2, Z)
S3 = segment (n, f)
R = meet (S3, Z)
gamma = < (N, O, R)

gamma = 27.692 344 071 228 924 º 13 *
gamma = 360.000 472 925 976 013º
Rel. Error = 0.000 131 368 326 7 %
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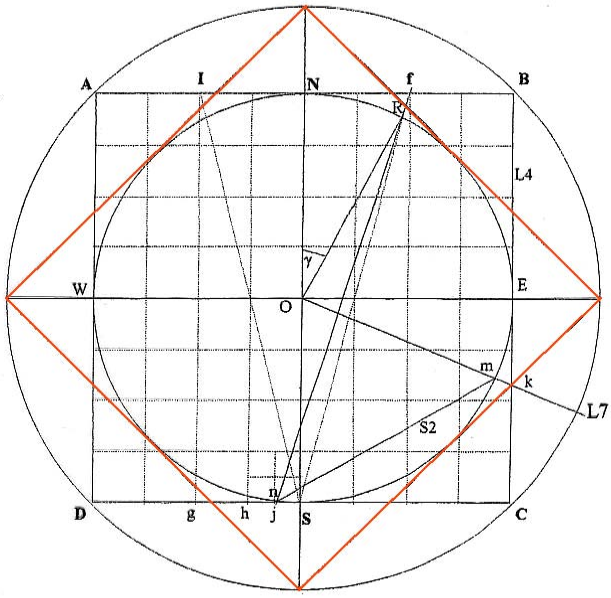

Base Figure and gamma

Vanessa added the second square in red to create the octagon, a geometrical form underlying much of the geometry of Chartres Cathedral. It is possible that the Medieval designers used a 13 pointed star to create the labyrinth, dividing the circle into 13 parts in this way,

Geometry construction by Lee Dickey, drawings by Vanessa Compton, provocation by Keith Critchlow and Ben Nicholson

References:

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Source:

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